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FUNDAMENTAL STATISTICAL IDEAS AS RELATED TO EXPLOSIVE SENSITIVITY TESTS

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Some of the limitations which must be kept in mind in considering results obtained from Bruceton up-and-down sensitivity tests are discussed. It is hoped that the discussion can be followed by those who have little or no background in the use of statistics. An understanding of these limitations should be helpful not only in the interpretation of results but also in planning the tests to be made. The work was carried out under Task Assignment NO 301/664/43006/12040.

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By direction

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FUNDAMENTAL STATISTICAL IDEAS AS RELATED TO EXPLOSIVE SENSITIVITY TESTS

Introduction

In order to understand the limitations of data obtained in sensitivity measurements of explosives one must be acquainted to some extent with certain fundamental ideas which concern the measurement of samples and the conclusions which may or may not be drawn from these measurements. Without such knowledge it is impossible to say what conclusions may or may not be drawn from the reported results as obtained from a given sample of explosive.

Before taking up the sensitivity measurement problem let us first consider an example of another type of measurement. Let us suppose that we wish to measure the height of Chinese so that we could compare this height with the height of some other group. Anyone considering this problem is immediately aware of several facts. One of these is that it is impracticable, if not impossible, to measure the height of all Chinese. Another is that not all Chinese have the same height so that, even after many of them have been measured, it is impossible to state the height of one who has not been measured. The best we could do would be to make an intelligent guess. The function of statistics in such a problem is to make the guess intelligent and to afford an estimate of the uncertainty of that guess.

Having decided that we are not going to measure the heights of all Chinese we take instead a sample from the entire group or population of Chinese. We then proceed to make the desired measurement of each individual of our sample. This sample should be one which is truly representative of the entire population if the results are to be accepted as typical. We should select the sample in such a way that every individual in the entire population has an equal chance of being chosen for measurement. It is very often difficult, if not impossible, to satisfy this requirement. This must always be considered in the interpretation of any set of measurements.

Statistical Parameters

The results of measurements taken on individuals of the sample are different values, one for each individual. This information will be in a more usable form if it can be condensed or summarized in some way. This is usually done by giving certain facts about the observed set of measurements. The simplest way in which these measurements can be characterized is by giving one number as a typical measurement and another to indicate the spread or dispersion of the data.

The best known of the values selected as typical measurements is the average or mean value. This is ordinarily obtained by taking the sum of all the measurements and dividing by the number of measurements. Other values which are used in this connection are the median and the mode. The median is that value which is so chosen that there are as many values greater than the median as there are less than its value. It is sometimes referred to as the fifty per cent point. Other values, such as the seventy-five per cent point or the ninety per cent point are also of interest at times. The mode is that value which is most frequently observed in the sample. If the distribution of the data is symmetrical or, in other words, if there is no tendency for measurements which differ from these typical values to be predominantly large or small, then the mean, median, and mode will be the same. However, in the usual case there will be some difference between them.

The simplest way to indicate the spread of the data is by giving the largest and smallest measurements obtained. The difference between these is known as the range. If this is given we should also give the number of observations since the range will increase as we take more and more readings. It is not difficult to see that the range is not entirely satisfactory since it tells nothing about the distribution of the intermediate values but depends only upon the two extremes. In order to give as complete a picture as possible every measurement should be considered in the measure of the dispersion. If we take the difference between each observation and the mean of all observations, square these differences, add, and divide by one less than the number of observations we obtain a measure of the dispersion which is known as the variance. Taking the square root of this quantity we would obtain a value which is very frequently used and which is known as the standard deviation. If the mean and standard deviation are known and if the variations of single observations from the mean are such as might be due entirely to random causes then any other information which we wish to know such as the ninety per cent point can be computed.

Of course, we would like to know the values associated with the population as a whole. However, as pointed out above, this is ordinarily impossible and we can only measure these values for the particular sample which we have selected for measurement. These sample values will differ from the true values and will, of course, differ from one sample to another. The sample values are estimates of the true values and a person who is familiar with statistics can tell something about how good these estimates are. In order to distinguish between the true value and its estimate many writers in statistics use different symbols for these quantities. The mean and standard deviation of the entire population are represented by μ and σ, while those of the measured sample are represented by m and s. If we were to take several samples and for each of these find the values of m and s we would have a set of values for each of these. For each of these sets we could again find a mean and a standard deviation. We could, for instance, obtain in this way a standard deviation of the mean of several samples. This is usually represented symbolically by s_m. It is not actually necessary to take several samples in order to get the values of such quantities as the standard deviation of the mean. They can be predicted from the information obtained from one sample provided this sample can be considered as representative.

Significant Differences

A question which must be answered very frequently is that of determining whether measurements taken with two or more samples can be said to indicate any real difference between these samples or whether, onthe other hand, the observed differences may not be due entirely to accidental differences between different samples taken from the same main population. As an example of this problem we may consider the desensitization of a given explosive. Suppose that several methods have been tried and sensitivity measurements have been made of the resulting material. A consideration of the means of these samples together with their standard deviations will give an indication of whether any of these samples has a sensitivity which is really less than that of the others. Suppose, for example, that impact sensitivities of eight samples had been measured as described in reference (a) and that the fifty per cent values were 31.4 cm, 30.3 cm, 35.2 cm, 32.6 cm, 28.0 cm, 35.2 cm, 32.6 cm, and 31.6 cm. Are these differences due to the way in which the sample was chosen or do they indicate actual differences in the sensitivities of the explosive mixes from which the samples were taken? To answer this question we would compare these differences with their standard deviations. If we find that the observed means differ more than could reasonably be attributed to chance we will say that there is a real difference or, as it is often stated, that the differences are statistically significant. It is worth noting that this is not the same thing as saying that the differences are of any practical importance. The question of the significance of observed differences such as these will be discussed later in this report.

Yes-No Tests

Up to this point we have been considering our problem as though every measurement gave us a value which could be regarded as the true value for the individual being measured. The height of an individual Chinese can be measured with no uncertainty but that due to carelessness of the person making the measurement or lack of precision of the instruments used. This is not, in general, true of sensitivity measurements. Here the best we can usually do is to determine for each item whether its sensitivity is greater or less than a certain selected value. Suppose, for example, that we were attempting to determine the sensitivity of flies to a certain poison when used in different dilutions or doses. The best we can do is to give each fly a certain dose of the poison and note the result. If the fly dies we know the dose was more than it could stand but we cannot try a smaller dose on the same fly. If the fly lives it is still not possible to test it again with a larger dose since the first dose has probably had some effect on the fly which would change its sensitivity characteristics. The situation in explosive sensitivity testing is the same. The item being tested will either be destroyed or at least damaged by the first attempt so that further testing on the same individual cannot be carried out.

There are two general types of testing methods which are in common use in a situation of this kind. In either case the investigator selects a series consisting of several different intensities of the stimulus in which he is interested. This may be the amount of the poison given the fly or the intensity of the shock to which an explosive sample is subjected. These stimulus intensity levels are spaced in some convenient way, quite frequently at equal intervals. From this point on the methods of testing differ markedly from each other. In one method the investigator decides to test a large number of individuals at each intensity level and note the number or the per cent of the individuals which respond to the stimulus, i.e., the flies which die or the items which explode. These results can be used to estimate the mean and standard deviation for the sample as a whole. One way of doing this is to plot the results on a special graph paper which is available for this purpose. This method of procedure is frequently used by the biologist since it is usually convenient to test many flies at one time. It is not often used in explosive sensitivity testing as it costs too much in terms of time and money. Descriptions of the method of analysis of data obtained in this way along with tables to be used in this analysis are given in references (b) and (c).

The other general testing procedure is one which is generally called an up-and-down method of testing. In using this method we would test an individual or possibly a small group of individuals at some intensity level. If the item, or group of items, responds to the stimulus at this level the next trial is made at the next lower stimulus level. If a failure to respond is observed the next test is made at the next higher level. This type of test gives an appreciable amount of information with the expenditure of a minimum amount of material. Predictions made on the basis of results obtained by this procedure are not as reliable as those obtained by the other method since more assumptions must be made in arriving at these predictions. The fact that one does not know at what level to make a trial until the preceding trial has been made and evaluated is sometimes a serious disadvantage.

One feature of the up-and-down type of test procedure is that the testing range is concentrated in one part of the entire possible range. Certain procedures make most of the trials near the eighty or ninety per cent points or near the ten or twenty per cent points. If a change in sensitivity level is made after each trial depending upon the response obtained in that trial the test will be concentrated near the fifty per cent point. This type of test was developed at Bruceton, Pennsylvania by the Explosives Research Laboratory and is commonly called the Bruceton up-and-down method of testing. A description of this type of test, together with the method of analysing the data obtained, is given in references (c) and (d). This is the method used for almost all testing done by the Explosives Sensitivity Measurement group of the Explosives Properties Division.

The analysis of the data obtained in a Bruceton up-and-down test gives estimates of the fifty per cent point and the standard deviation of a single observation. Combining these two estimates we can get estimates of other per cent points such as the ninety per cent point. The standard deviation of the mean can also be estimated. This is needed if we wish to compare one sample with another. The method of making this comparison is described in reference (d). The precision of these estimates is affected by several factors among which are the size of the sample, the way in which the sensitivity levels are spaced, and the way in which the sample is selected. These will be discussed in succeeding paragraphs.

Effect of Sample Size

The effect of the size of the sample upon the accuracy of the results is not difficult to see when we consider a few simple facts. Probably everyone would agree that results obtained from measurements made on a sample of only one individual could not be expected to give a very good basis for predictions about the other unmeasured items. If, on the other hand, the sample is enlarged until it includes the entire population then we will have learned all that it is possible to learn and the uncertainty will have been reduced to a minimum. Increasing the sample size, then, increases the reliability which can be placed in the results obtained. As we increase the sample size we should find, if we analyse the results from time to time, that the mean and standard deviation remain essentially constant while their standard deviations decrease steadily. It is clear that this should be so when we remember that m and s are estimates of the fixed unknown quantities μ and σ, while s_m and s_s can be regarded as measuring the uncertainty of m and s. It can be proved that in order to decrease s_m by a factor of two the sample size must be made four times as great.

In order to illustrate this effect of sample size an example was worked the results of which are given in Table I. This example deals with data which might have been obtained in the measurement of explosive impact sensitivity on the NOL impact machine as described in reference (a). Different per cent points for the sensitivity distribution were computed for tests of twenty-five, fifty, and one hundred trials. For each test it was assumed that the mean was 1.5051 on the logarithmic scale, equivalent to 32 cm, with a standard deviation of 0.125 units. The tabulated values are, in each case, obtained by taking the true value of the per cent point, adding and subtracting its standard deviation, and changing the results to the corresponding heights in centimeters. The chance that a particular value will fall within this range is about two out of three. Notice that the uncertainty associated with the one-tenth per cent point decreases from ten centimeters for twenty-five trials to five centimeters for one hundred trials.

Effect of Step Size

In a Bruceton up-and-down test the sensitivity levels are chosen as equally spaced according to some scale which, in explosive sensitivity testing, is commonly logarithmic. The difference between any two adjacent levels is known as the step size. The choice of a small step size will tend to concentrate the trials near the fifty per cent point. The choice of a larger size will spread out the trials into the region of the ten and ninety per cent points. Any test of this sort will give more reliable results in the region which is being tested. This indicates that the Bruceton method will give a better determination of the fifty per cent point if a small step size is used and better determinations of the ten and ninety per cent points if larger steps are used. This is illustrated in Table II which was constructed in the same way as Table I to show the effect of step size. Notice that the indicated spread for the fifty per cent point increases as the step size is increased while the spreads for the other points decrease. In computing the values given in Table I and Table II it has been assumed that the frequency follows the so-called normal distribution. If this is not true the choice of the size of the step will have a greater effect than that indicated in Table II.

Effect of Non-normality of the Distribution

The actual frequency distribution of the individual sensitivities may differ from the assumed normal form in two ways. In a skew distribution the extreme values are unequally distributed with respect to the central values. In this case the mean will differ from the fifty per cent point and will be nearer the side having the more extreme values. In the other case the distribution will be either flatter or more peaked than the normal form. In this case the mean and median agree but the standard deviation obtained will depend upon the region in which measurements are made since it differs in the different parts of the frequency distribution. Reference (e) gives some results which were obtained empirically with two selected distributions and which illustrate the effect of non-normality on estimates of such values as the ninety per cent point. These estimates may be either too large or too small depending upon how the distribution is skewed. The estimated fifty per cent point will be between the true fifty per cent point and the mean. A small step size will give an estimate which is near the true fifty per cent point. As the size of the step is increased this estimate will approach the value of the mean. This can be illustrated by using the population (A) of reference (e) which is one in which most of the individuals are concentrated near the upper values with a few scattering extreme low values. The mean is 21.26, the fifty per cent point is 23.1, and the standard deviation is 5. Using the most probable values for a test of twenty-two trials with a step of two units we obtain $m = 22.64$ and $s = 3.22$. For the same size step and a test of fifty trials we get $m = 22.44$ and $s = 4.80$. Fifty trials with a step size of five units gives $m = 21.90$ and $s = 4.95$. Notice also that increasing the size of the step or the number of trials gives values of s which are better estimates of σ than the estimate of 3.22 obtained with the short test with a small step size.

Effect of Random Selection

In order to illustrate the change which may be expected to occur from one sample to another, as the result of chance variation of selection, several samples were taken from the same known population. A table of random numbers was used for this purpose. This table had been made up by taking numbers which form a normal frequency distribution with a known mean and standard deviation and then arranging them in a random order. By choosing numbers from such a table in any predetermined order we get a random sample from a population whose mean and standard deviation are known. These values were chosen as representative of those which are obtained in measurements of explosive impact sensitivity on the impact machine used at the Naval Ordnance Laboratory. They were $\mu = 1.505$ and $\sigma = 0.125$. In these sensitivity measurements the height is measured on a logarithmic scale so the mean obtained is the logarithm of the mean height in centimeters. Thus the value 1.505 corresponds to a height of 32 centimeters. A random sample consisting of one hundred numbers was selected and used to represent trials in a Bruceton up-and-down impact sensitivity test. Since the actual value is known for each number drawn, the true mean and standard deviation for the sample can be found. These values turn out to be $m = 1.497$ and $s = 0.121$. Comparison of these values with the estimates obtained from the Bruceton test analysis to be presented later will give a meausre of the uncertainty introduced by the fact that the Bruceton technique is a yes-no type of test.

The sample taken from the population of random numbers was used for four Bruceton type tests. The records of the individual observations are shown in Figures 1 through 4. The results are shown in the form of a chart with each line representing a height at which a trial was made. An explosion is represented by an X and a non-explosion by an O. These tests can be subdivided into tests of fifty or twenty-five trials. This was done and the computations made for each test and subtest. Trials at the beginning of a test which occur before a reversal between an explosion and a non-explosion are not counted in the computations. The results are given in Tables III and IV. In making the tests given in Figures 1 and 2 the step size was that ordinarily used at the Naval Ordnance Laboratory. For the tests shown in Figures 3 and 4 the size of the step was halved.

Since the level at which any trial in a Bruceton type test is made will depend upon the results of the previous trial the order in which the individual items are drawn will have an effect upon the result of the test. Tests (A) and (B), shown in Figures 1 and 2, are with the same individuals and differ only in the order in which these were tested. The same statement is true of tests (C) and (D) which are shown in Figures 3 and 4. Comparing the results of test (A) with those of test (B) and those of test (C) with the results of test (D) we can see an example of the effect of order in the selection of items for a Bruceton type test.

Remembering that the fifty per cent point is known to be at a height of 32 centimeters we can see some interesting results of chance variation. The third subdivision of twenty-five trials from test (A) has nine trials made at 32 centimeters and of these only two are explosions. The second and third groups of twenty-five from test (C) indicate a lower height. In the second group there are five explosions in six trials at 28.5 centimeters and in the third group there are seven explosions in nine trials at 32 centimeters. The first group of test (D) gave seven explosions in eight trials at 32 centimeters.

Comparison of Samples

The eight impact sensitivities of different samples in the illustration given earlier are actually the eight values obtained by subdividing tests (A) and (B) into groups of twenty-five trials each. Since we know that these are the results of tests made on individuals of the same population we can answer the question asked earlier by saying that the differences indicated are not real but are in fact due to the way in which the samples were chosen.

In actual practice we usually would not know that our samples were all from the same population. In this case we can use a method discussed in reference (d) for comparing several samples to determine whether or not it is reasonable to suppose that the observed differences are due to the random variation involved in the sample selection. This method is a slight modification of the statistician's Chi square test. To use this test we compute a number which, in this case, can be considered as a measure of the departure from uniformity between the sample means and which in this case gives us the value 5.3. It can be determined by the use of certain tables that once in twenty times this value will be as large as 15.3 due to chance variation alone and that it will be as large as 12 once in ten times. Since our value of 5.3 is considerably smaller than these we see that it is not at all unlikely that the observed differences are due to chance. We can say therefore that the observed differences in sensitivities do not necessarily represent real differences. Notice that we do not say that there is no difference. We can only say that we have not shown that a difference exists. This is similar to the verdict of a jury which finds the defendant not guilty. The real meaning of this verdict is that the defendant has not been proved to be guilty which is quite a different thing from saying that he is innocent.

Conclusion

Some of the problems which confront one who wishes to measure the sensitivity of an explosive are discussed in this report. Some of the fundamental terms used in this measurement have been explained. The effects of some of the factors, such as the size and method of selection of the sample, have been discussed and illustrated. Problems of comparing the data from any given sample with that from other samples or with previous standards have been mentioned. Some of these factors are under the control of the experimenter and knowledge of their effects should form a part of the basis of his judgment in the selection of the sample to be tested. An understanding of all these things, whether under the control of the experimenter or not, is necessary to an intelligent interpretation of the resulting data.

Table I

Effect of Sample Size on Bruceton Results

	25 Shots	50 Shots	100 Shots
0.1% Point	9.0 - 19.2	10.1 - 17.2	10.9 - 15.9
1.0% "	12.2 - 21.9	13.3 - 20.1	14.2 - 19.0
5.0% "	16.1 - 24.7	17.1 - 23.2	17.9 - 22.2
10.0% "	18.6 - 26.3	19.6 - 25.0	20.3 - 24.1
50.0% "	29.5 - 34.6	30.3 - 33.8	30.8 - 33.3
90.0% "	38.9 - 55.1	40.9 - 52.3	42.4 - 50.5
95.0% "	41.5 - 63.6	44.2 - 59.7	46.2 - 57.2
99.0% "	46.7 - 83.6	50.9 - 76.8	54.0 - 72.3
99.9% "	53.2 -113.9	59.5 -101.9	64.4 - 94.2

Table II

Effect of Step Size on Bruceton Results

	Half Size	Regular Size	Double Size
0.1% Point	10.3 - 16.8	10.9 - 15.9	11.2 - 15.5
1.0% "	13.6 - 19.7	14.2 - 19.0	14.4 - 18.6
5.0% "	17.4 - 22.8	17.9 - 22.2	18.1 - 21.9
10.0% "	19.9 - 24.6	20.3 - 24.1	20.4 - 23.9
50.0% "	30.8 - 33.2	30.8 - 33.3	30.6 - 33.5
90.0% "	41.6 - 51.5	42.4 - 50.5	42.8 - 50.1
95.0% "	44.9 - 58.7	46.2 - 57.2	46.7 - 56.5
99.0% "	51.9 - 75.2	54.0 - 72.3	55.1 - 70.9
99.9% "	61.0 - 99.4	64.4 - 94.2	66.1 - 91.7

Table III

Simulated Drop Tests Using Random Numbers
Regular Size Step

	HEIGHT log	HEIGHT cm	100 Shots	50 Shots		25 Shots			
	log	cm	(X) (O)	(X) (O)	(X) (O)	(X) (O)	(X) (O)	(X) (O)	(X) (O)
Test (A)	1.805	64.0	1 - 0		1 - 0			1 - 0	
	1.705	50.5	5 - 1	1 - 0	4 - 1	1 - 0		2 - 1	2 - 0
	1.605	40.5	21 - 5	9 - 2	12 - 3	4 - 1	4 - 1	7 - 1	5 - 2
	1.505	32.0	19 - 21	12 - 10	7 - 11	6 - 5	6 - 5	2 - 7	5 - 4
	1.405	25.5	4 - 19	2 - 12	2 - 7	1 - 6	1 - 6	1 - 2	1 - 5
	1.305	20.0	0 - 4	0 - 2	0 - 2	0 - 1	0 - 1	0 - 1	0 - 1
Mean, cm.			32.7	31.1	33.9	31.4	30.3	35.2	32.6
log			1.515	1.493	1.530	1.497	1.482	1.547	1.513
s			0.124	0.086	0.149	0.107	0.072	0.165	0.136
s_m			0.017	0.018	0.029	0.030	0.024	0.045	0.041
	HEIGHT log	HEIGHT cm	(X) (O)	(X) (O)	(X) (O)	(X) (O)	(X) (O)	(K) (O)	(K) (O)
Test (B)	1.705	50.5	7 - 0	4 - 0	3 - 0	1 - 0	3 - 0	1 - 0	2 - 0
	1.605	40.5	16 - 8	7 - 4	9 - 4	2 - 1	5 - 3	6 - 1	3 - 3
	1.505	32.0	18 - 17	9 - 7	9 - 10	5 -. 2	4 - 5	5 - 6	4 - 4
	1.405	25.5	8 - 18	5 - 9	3 - 9	5 - 4	0 - 4	1 - 4	2 - 5
	1.305	20.0	0 - 8	0 - 5	0 - 3	0 - 5		0 - 1	0 - 2
Mean, cm			31.3	31.3	32.0	28.0	35.2	32.6	31.6
log			1.496	1.495	1.505	1.447	1.547	1.513	1.500
s			0.166	0.167	0.132	0.165	0.107	0.107	0.178
s_m			0.023	0.032	0.026	0.047	0.031	0.031	0.051

Table IV

Simulated Drop Tests Using Random Numbers
Half Size Step

	HEIGHT log	HEIGHT cm	100 Shots (X) - (O)	50 Shots (X) - (O)	50 Shots (X) - (O)	25 Shots (X) - (O)	25 Shots (X) - (O)	25 Shots (X) - (O)	25 Shots (X) - (O)
Test (C)	1.605	40.5	4 - 0	3 - 0	1 - 0	2 - 0	1 - 0		1 - 0
	1.555	36.0	12 - 4	5 - 3	7 - 1	4 - 2	1 - 1	2 - 0	5 - 1
	1.505	32.0	18 - 13	5 - 5	13 - 8	4 - 5	1 - 0	7 - 2	6 - 6
	1.455	28.5	8 - 19	6 - 5	2 - 14	1 - 4	5 - 1	2 - 7	0 - 6
	1.405	25.5	7 - 8	6 - 6	1 - 2	1 - 1	5 - 5	1 - 2	
	1.355	22.5	0 - 7	0 - 6	0 - 1	0 - 1	0 - 5	0 - 1	
Mean, cm			30.1	29.2	30.9	31.7	27.2	29.6	32.3
log			1.478	1.466	1.490	1.501	1.434	1.472	1.509
s			0.110	0.154	0.117	0.112	0.112	0.059	0.039
s_m			0.015	0.023	0.023	0.030	0.030	0.017	0.012
	HEIGHT log	HEIGHT cm	(X) - (O)	(X) - (O)	(X) - (O)	(X) - (O)	(X) - (O)	(X) - (O)	(X) - (O)
Test (D)	1.655	45.0	1 - 0		1 - 0			1 - 0	
	1.605	40.5	6 - 1	3 - 0	3 - 1		3 - 0	3 - 1	
	1.555	36.0	11 - 7	5 - 4	6 - 3	1 - 0	4 - 4	6 - 2	0 - 1
	1.505	32.0	16 - 12	9 - 6	7 - 6	7 - 1	2 - 5	4 - 5	3 - 1
	1.455	28.5	10 - 16	6 - 9	4 - 7	5 - 6	1 - 2	0 - 3	3 - 4
	1.405	25.5	4 - 10	1 - 6	3 - 4	0 - 5	1 - 1		3 - 4
	1.355	22.5	1 - 4	0 - 1	1 - 3		0 - 1		1 - 3
	1.305	20.0	0 - 1		0 - 1				0 - 1
Mean, cm			30.6	30.6	30.5	29.0	32.5	34.3	27.2*
log			1.485	1.486	1.484	1.463	1.512	1.535	1.435*
s			0.134	0.092	0.176	0.037	0.136	0.075	0.155*
s_m			0.018	0.018	0.033	0.011	0.038	0.022	0.043*

*These values were computed by assuming a non-existant explosion at 40.5 cm.

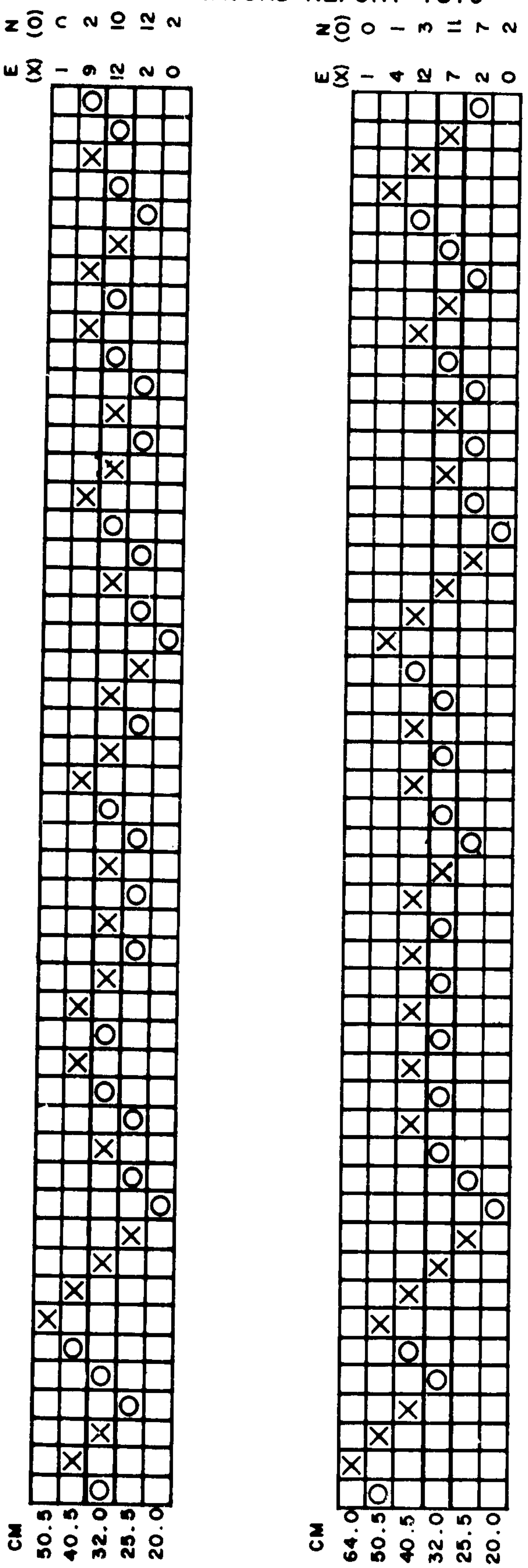

FIG.1 DROP TEST A

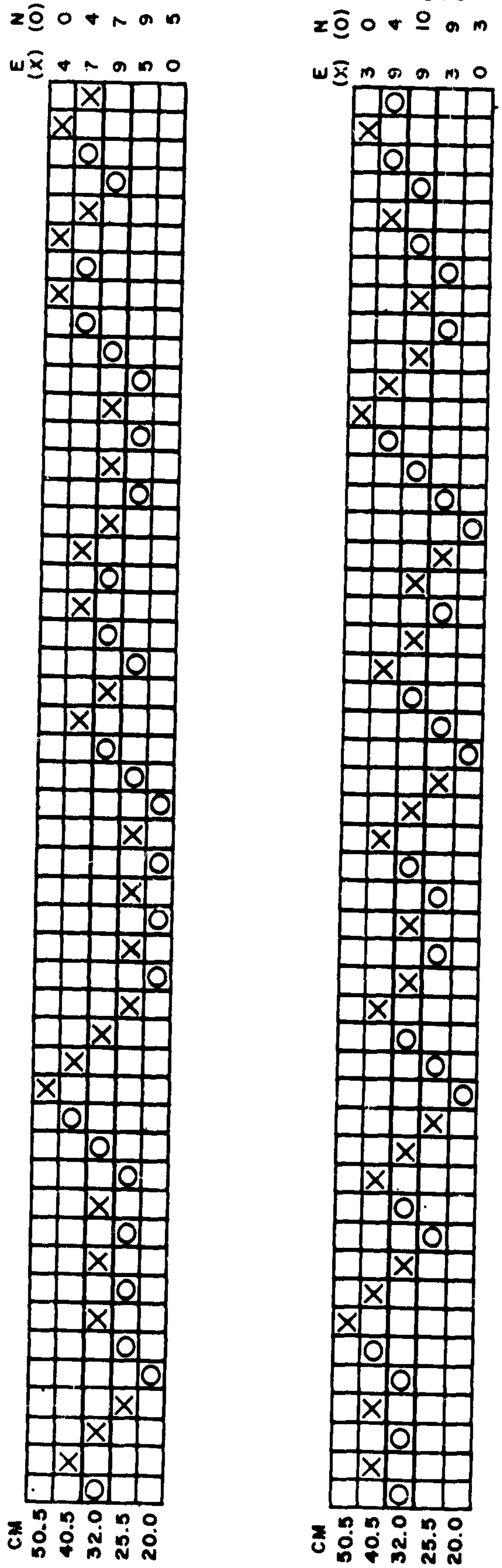

FIG.2 DROP TEST B

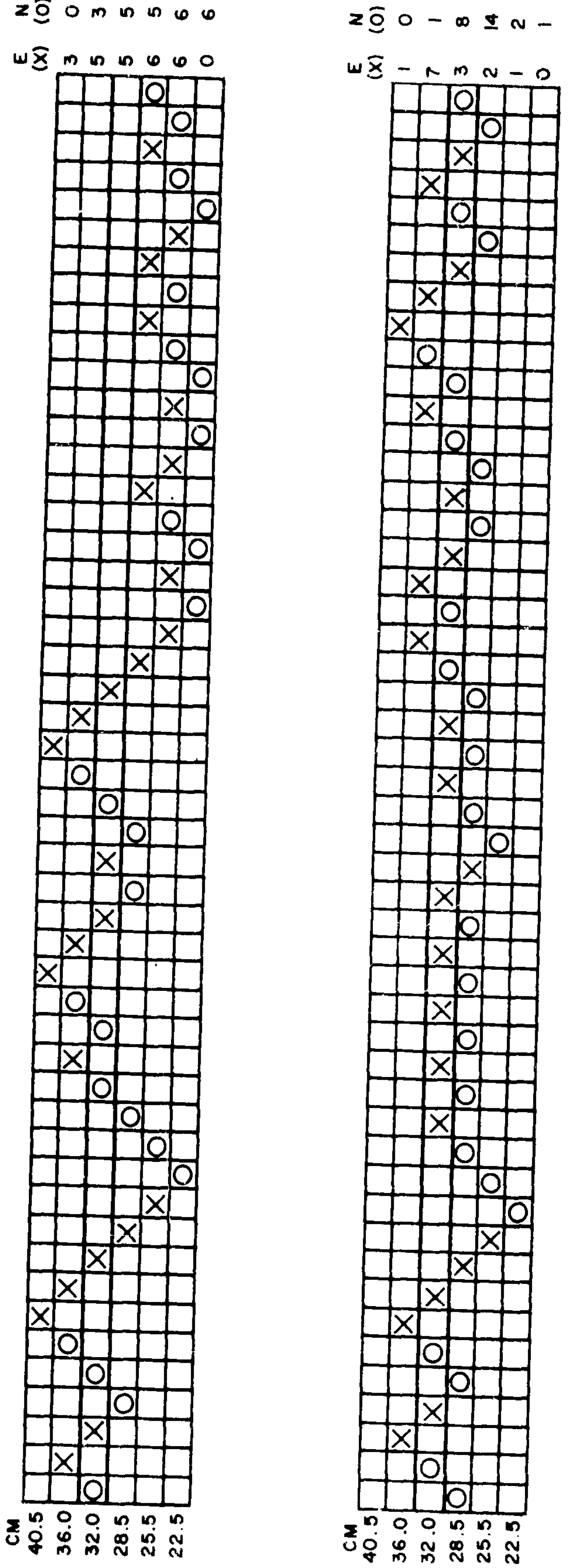

FIG.3 DROP TEST C

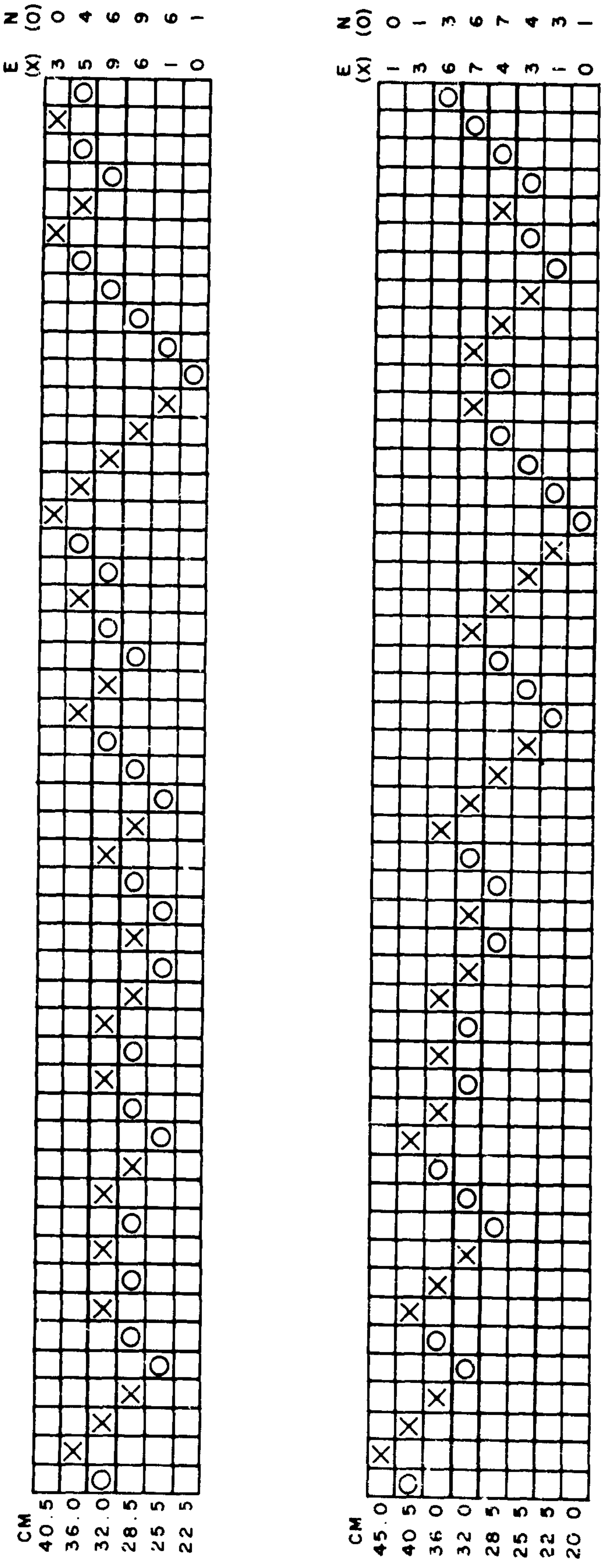

FIG. 4 DROP TEST D

References

(a) NOLM 10,003, Studies of the ERL Type 12 Drop-Weight Imp^ct Machine at NOL, E.H. Eyster, L.C. Smith, 25 Jan 1949

(b) "Probit Analysis -- A Statistical Treatment of the Sigmoid Response Curve", D.J. Finney, Cambridge University Press, 1947

(c) NAVORD Report 2101, Statistical Methods Appropriate for Evaluation of Fuze Explosive Train Safety and Reliability, H.P. Culling, 13 Oct 1953

(d) AMP Report 101.1R, Statistical Analysis for a New Procedure in Sensitivity Experiments

(e) NAVORD Report 2129, Bruceton Test Results with Non-Normal Populations, L.D. Hampton, 26 Jun 1951

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